LECTURE X

DISTANCES AND COSMOLOGICAL OBSERVATIONS

PROPER DISTANCE

DISTANCE TO AN OBJECT AT TIME t

ACTUAL RADIAL DISTANCE

(SIMULTANEOUS MEASUREMENT)

$$D_p = \int_0^r \sqrt{g_{rr}}\, dr = a(t)\, r$$

(IN THIS METRIC: PROPER DISTANCE $\hat{=}$ COMOVING DISTANCE)

PROPAGATION OF PHOTON: $ds^2 = 0$

$$\Rightarrow \int_r^0 dr = \int_{t(z)}^{t_0} \frac{c\,dt}{a(t)}$$

$$-r = \int_{t(z)}^{t_0} \frac{c\,dt}{a(t)} = -\int_z^0 \frac{c\,dz}{H(z)}$$

$$D_{p,0} = r = \int_0^z \frac{c\,d\tilde{z}}{H(\tilde{z})}$$

$$D_p(t) = a(t) \cdot D_{p,0} = a(t) \cdot r$$

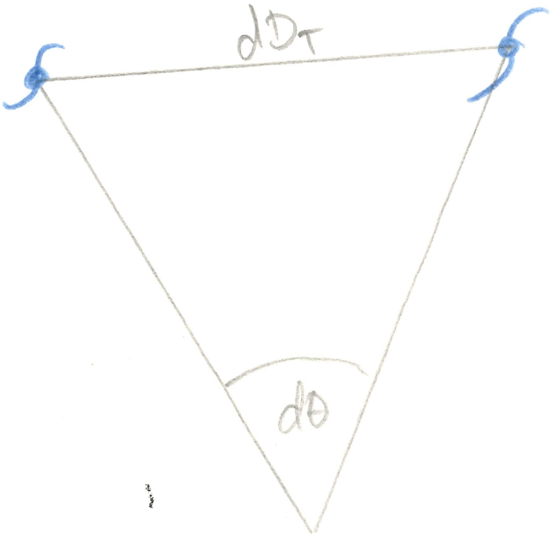
REQUIRE $H(z)$

$$\Rightarrow \quad H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{kc^2}{R_{c,0}^2 a^2}$$

$$\frac{8\pi G}{3}\rho = H_0^2\left(\Omega_{m,0} a^{-3} + \Omega_{r,0} a^{-4}\right)$$

$$H(z) = H_0\left[\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0} + \Omega_{k,0}(1+z)^2\right]^{1/2}$$

TRANSVERSE PROPER DISTANCE



$$dD_T = a(t)\, R_{c,0}\, S_k\!\left(\frac{D_{P,0}}{R_{c,0}}\right) d\theta$$

(USE $D_{P,0} = r$)

USE : $D_P(t) = a(t)\, D_{P,0}$

$R_c(t) = a(t)\, R_{c,0}$

assume : $dt = 0$

ANGULAR DIAMETER DISTANCE

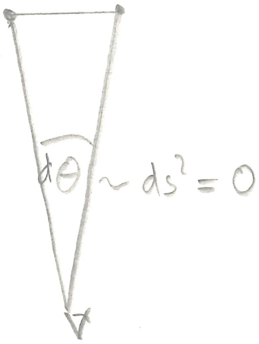
DEFINITION PER OPERATION:

$$dl = D_\theta \cdot d\theta$$

(dl ↑ PHYSICAL DIAMETRE; $d\theta$ ↑ ANGLE)

IF UNIVERSE DOES NOT EXPAND:

$$dl = R_c\, S_k\!\left(\frac{D_P}{R_c}\right) d\theta$$



ANGLE DOES NOT CHANGE WITH EXPANSION

PROPER TRANSVERSE SIZE AT EMISSION

$$D_p(t_e) = a(t_e) R_{c,0} S_k\left(\frac{r\,a(t_e)}{R_{c,0}\,a(t_e)}\right) d\theta \overset{!}{=} dl$$

$$= \frac{1}{1+z} R_{c,0} S_k\left(\frac{D_{p,0}}{R_{c,0}}\right) d\theta$$

$$D_\theta = \frac{R_{c,0}}{1+z} S_k\left(\frac{D_{p,0}}{R_{c,0}}\right)$$

SLIDE - 87 (78)

LUMINOSITY DISTANCE

LUMINOSITY DISTANCE:

INTRINSIC LUMINOSITY : L

RECEIVED FLUX : $\mathcal{F}$

$$\mathcal{F} = \frac{L}{4\pi D_L^2}$$

a) PHOTONS RECEIVED BY AN OBSERVER DISTRIBUTED OVER SPHERE WITH RADIUS $D_{p,0}$; SURFACE AREA :

$$\int_{4\pi} R_{c,0}^2 S_k^2\left(\frac{D_{p,0}}{R_{c,0}}\right) \sin\theta \, d\phi \, d\theta = 4\pi R_{c,0}^2 S_k^2\left(\frac{D_{p,0}}{R_{c,0}}\right)$$

b) ENERGY OF EACH PHOTON
REDUCED BY REDSHIFT EFFECT $\frac{1}{1+z}$

$\Rightarrow$

FLUX " " " " "

c) CLOCKS AT DISTANT GALAXY REDUCED BY FACTOR $(1+z)$ $\Rightarrow$ REDUCED RATE AT OBSERVER BY $\frac{1}{1+z}$

b) + c) : $P_{obs} = \frac{1}{(1+z)^2} P_{em}$

$$P_{em} = \frac{h\nu_e}{\delta t_e}$$

$$\Rightarrow \quad F = \frac{L}{4\pi (1+z)^2 R_{c,0}^2 S_k^2 \left(\frac{D_{P,0}}{R_{c,0}}\right)}$$

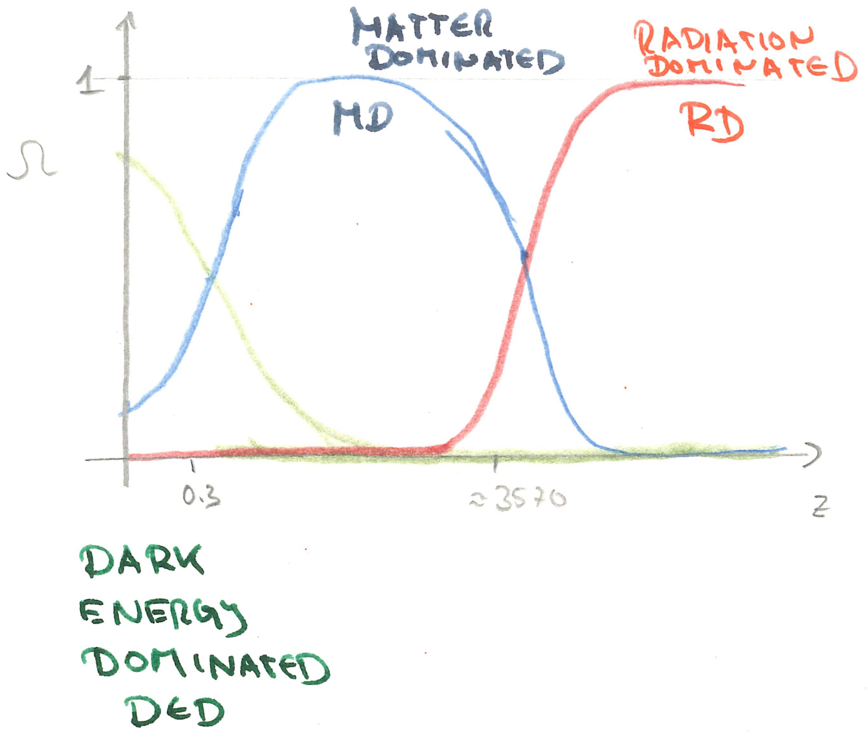
$$D_L = (1+z) R_{c,0} S_k \left(\frac{D_{P,0}}{R_{c,0}}\right) = (1+z)^2 D_\theta$$

SLIDE - 88 - 91 (79 - 82)

$(m = M + 5 \log_{10} d_L + 25)$ (d_L in Mpc)

THE EARLY UNIVERSE

DEFINE: $\Omega_{tot}(z) = \Omega_m(z) + \Omega_\Lambda(z) + \Omega_r(z)$



RADIATION
"DARK" MATTER
COSMOLOGICAL CONSTANT

EARLY UNIVERSE: PARTICLES AND THEIR ANTI-PARTICLES ARE CREATED AND ANNIHILATED IN EQUILIBRIUM WITH PHOTONS AND OTHER PARTICLES

- PARTICLE SPECIES DISAPPER WHEN TEMPERATURE FALLS BELOW REST MASS

BUT PARTICLES AND ANTI-PARTICLE DENSITY ARE NOT EQUAL (C, CP - VIOLATION)

⇒ NOT ALL DISSAPEAR

⇒ PROTONS, ELECTRONS, NEUTRONS SURVIVE

RATIO OF BARYONS - TO - PHOTONS TODAY

$1 : 10^9$

(TINY INITIAL ASSYMETRY)

PRIMORDIAL NUCLEOSYNTHESIS

AT $kT = 10\,\mathrm{MeV}$ ($\approx 10^{10}$K) : RD

WEAK INTERACTION RATES > EXPANSION RATE OF UNIVERSE

EQUILIBRIUM: $p, n, e^-, e^+, \nu, \bar{\nu}$

$$\frac{N_n}{N_p} = \exp\left(-\frac{m_n - m_p}{kT}\right) = \exp\left(-\frac{1.293\,\mathrm{MeV}}{kT}\right)$$

$$e^+ + n \Leftrightarrow p + \bar{\nu}_e$$

$$\nu_e + n \Leftrightarrow p + e^-$$

$\Rightarrow$ AT 10 MeV: $\frac{n}{p} \sim 1$

WEAK INTERACTION RATES ($\nu_e + n$):

$$\tau_{WEAK} \sim (\sigma_{WEAK} N_\nu c)^{-1}$$

$\rho_\gamma = \sigma_{SB} T^4$
$\sim (1+z)^{+4} \sim R^{-4}$
$\Rightarrow T \sim R^{-1}$

WITH $\sigma_{WEAK} \sim E^2 \sim (kT)^2 \sim$

SINCE $N_\nu \sim R^{-3} \sim T^3$: $\tau_{WEAK} \sim T^{-5}$

AT $t \approx 1$ sec ; $T \sim 1$ MeV ; $\rho \sim 0.01 \frac{g}{cm^3}$

INTERACTION RATE BETWEEN n AND p $(1s)^{-1}$
SMALLER THAN EXPANSION RATE ($\sim 1s^{-1}$)

- WEAK INTERACTION CAN NOT KEEP EQUILIBRIUM

- RATIO OF PROTONS AND NEUTRONS FREEZES OUT
- NEUTRONS DECAY FURTHER ($\tau \approx 886s$)
- BY $\frac{N}{P} \sim \frac{1}{7}$ UNIVERSE COLD ENOUGH TO FORM DEUTIRIUM

$\Rightarrow$ REMAINING NEUTRONS FORM DEUTERIUM

$$p + n \Leftrightarrow D + \gamma$$
$$D + D \Leftrightarrow {}^3He + n \Leftrightarrow {}^3H + p$$
$${}^3He + D \Leftrightarrow {}^4He + n$$

(FURTHER: ${}^7Li$)

× NO HEAVIER ELEMENTS

- NO STABLE ISOTOPES WITH MASS NUMBER 5 OR 8 (${}^8Be$ INSTABLE)
- TOO LOW DENSITY FOR TRIPLE-α TO FORM ${}^{12}C$
- DETERMINED BY PHOTON-TO-BARYON RATIO
- DEUTERIUM ABSORBTION IN HIGH-REDSHIFT QUASARS $\triangleq$ CMB VALUE

BIG BANG MODEL BETWEEN 1s AND 100,000 yrs SLIDE-93-95

VERY EARLY UNIVERSE

- VERY HIGH ENERGY DENSITIES
- SYMMETRIES NOT BROKEN
- STRONG, WEAK AND EM FORCES UNIFIED

TIME	TEMP.	ENERGY	BREAKING OF SYMMETRY
10^{-43} s	10^{32} K	10^{16} TeV	THEORY OF EVERYTHING
10^{-36} s	10^{28} K	10^{12} TeV	GRAND UNIFICAT.
10^{-12} s	10^{16} K	1 TeV	ELECTROWEAK UNIF.

10^{-43} s: QUANTUM MECHANICS AND GENERAL RELATIVITY IMPORTANT

COMPTON WAVELENGTH:

$$\lambda_{PL} = c\,t_{PL} = \frac{\hbar}{m_{PL}c}$$

WHAT MASS HAS SCHWARZSCHILD RADIUS EQUAL TO COMPTON WAVELENGTH

$$\frac{\hbar}{m_{PL}c} = \lambda_{PL} = r_s = \frac{G m_{PL}}{c^2}$$

$$\Rightarrow m_{PL} = \left(\frac{\hbar c}{G}\right)^{1/2} = 2.2\cdot 10^{-8}\,\text{kg} = 1.22\cdot 10^{19}\,\frac{\text{GeV}}{c^2}$$

PLANCK MASS

PLANCK LENGTH

$$\lambda_{pl} = \frac{G m_{pl}}{c^2} = 1.62 \cdot 10^{-35} m$$

$$t_{pl} = \frac{l_{pl}}{c} = 5.31 \cdot 10^{-44} s$$

BEFORE THIS CAN NOT USE GR

PROBLEMS OF THE STANDARD BIG BANG SCENARIO

1.) WHY DOES THE UNIVERSE HAVE A FLAT GEOMETRY? FLATNESS PROBLEM

2.) WHY DOES THE CMB HAVE THE SAME TEMPERATURE IN ALL DIRECTIONS? HORIZON PROBLEM

3.) WHY ISN'T THE UNIVERSE FULL OF MONOPOLES?

4.) WHERE DO THE PRIMORDIAL FLUCTUATIONS COME FROM? STRUCTURE FORMATION PROBLEM

5.) WHY DOES THE COSMOLOGICAL CONSTANT HAVE SUCH A SMALL VALUE?

THE FLATNESS PROBLEM

MATTER / RADIATION AND CURVATURE

$$1 = \Omega_{m,r} + \Omega_K = \Omega_m + \frac{kc^2}{R_{c,0}^2 a^2 H^2}$$

$$\Leftrightarrow \quad a^2 H^2 \Omega_{m,r} \left(\frac{1}{\Omega_{m,r}} - 1 \right) = \frac{kc^2}{R_{c,0}^2}$$

USE: $\Omega_{m,r} = \dfrac{8\pi G \rho_{m,r}}{3H^2}$

$$a^2 \rho_{m,r} \left(\frac{1}{\Omega_{m,r}} - 1 \right) = \frac{3kc^2}{8\pi G R_{c,0}^2}$$

$$\rho_{m,r} = \rho_0 (1+z)^{3 \dots 4} \qquad a = \frac{1}{1+z}$$

$$\frac{1}{\Omega_{m,r}} - 1 = \frac{3kc^2}{8\pi G \rho_0 R_{c,0}^2} \frac{1}{(1+z)^{1 \dots 2}}$$

USING EQN. FOR $z = 0$

$$\Rightarrow \quad \frac{1}{\Omega_{m,r}} - 1 = \left(\frac{1}{\Omega_{m,r,0}} - 1 \right) \cdot \frac{1}{(1+z)^{1 \dots 2}}$$

FOR $\Omega_{m} = 1$ TODAY Ω ALWAYS $\underline{1}$

FINE TUNING!

$z = 0$; $\Omega_m = 0.3$

$z = 1$; $\Omega_m = 0.47$

$z = 5$; $\Omega_m = 0.72$

$z = 10^3$; $\Omega_m = 0.998$

$z = 10^5$; $\Omega_m = 0.9998$

SLIDE-95