LECTURE II

FURTHER PROPERTIES OF SPIRAL GALAXIES

RADIUS-LUMINOSITY RELATION

$$\log_{10} R_{25} = -0{,}249\, M_B - 4{,}00$$

(R_{25} : SURFACE BRIGHTNESS 25 $\frac{\text{B-mag}}{\text{arcmin}^2}$)

INDEP. OF TYPE

MASSES AND M/L RATIO, COLOR

TULLY-FISHER AND RAD-LUM RELATION

$$M = \frac{V^2 R}{G}$$

$$10^9 M_\odot < M < 10^{12} M_\odot$$

FOR EARLY TYPE SPIRAL (ALMOST INDEP. OF TYPE)

$$Sa : \left\langle \frac{M}{L_B} \right\rangle = 6.2 \pm 0.6$$

$$Sb : \left\langle \frac{M}{L_B} \right\rangle = 4.5 \pm 0.4$$

$$Sc : \left\langle \frac{M}{L_B} \right\rangle = 2.6 \pm 0.2$$

↳ GREATER FRACTION OF MASSIVE MAIN SEQUENCE STARS

↳ BLUER THAN Sa AND Sb

Sa : $B-V = 0.75$

Sb : $B-V = 0.64$

Sc : $B-V = 0.52$

(Irr : $B-V = 0.4$)

BLUE MAIN SEQUENCE STAR SHORT-LIVED
↳ FORMED RECENTLY

Sc : GAS AND DUST ⇒ TO PRODUE BLUE STARS

FROM 21cm, $H\alpha$ AND CO (TRACER OF H_2)
GAS FRACTION INCREASES FROM Sa TO Sc

SUPERMASSIVE BLACK HOLES

- MOTION OF STARS AND GAS NEAR CENTRE OF SPIRALS

M31 : $\frac{M}{L} = 35 \frac{M_\odot}{L_\odot}$

⇒ A LOT OF NON-LUMINOUS MATTER IN SMALL AREA

M31 : TRIPLE NUCLEUS MOTION SLIDE-18

$\Rightarrow 1.4^{+0.9}_{-0.3} \times 10^8 M_\odot$

FROM VIRIAL THEOREM:

$$\frac{1}{2}\left\langle \frac{d^2 I}{dt^2} \right\rangle - 2\langle K \rangle = \langle U \rangle$$

kinetic potential

MOMENT OF INERTIA EQUILIBRIUM $\hat{=}$ 0

$$-2\langle K \rangle = \langle U \rangle$$

LARGE AMOUNT OF STARS: TIME AVERAGING

$\hat{=}$ AVERAGING OVER STARS

$$U = -2 \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2$$

$$U = -\frac{1}{2} \sum_i \sum_{j \neq i} \frac{G m_i m_j}{r_{ij}}$$

AVERAGE OVER LARGE COLLECTION OF STARS

$\langle v_r^2 \rangle \approx \langle v_\theta^2 \rangle \approx \langle v_\phi^2 \rangle$

$$M = Nm$$

$$-\frac{m}{N} \sum_{i=1}^{N} v_i^2 = \frac{U}{N}$$

$$\langle v^2 \rangle = \langle v_r^2 \rangle + \underbrace{\langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle}_{\text{Galaxy too far away to measure this}} = 3\langle v_r^2 \rangle$$

ONLY MEASURED

isotropy

$\sigma^2 = \langle (v^2 - \bar{v}^2) \rangle$

$\bar{v} = 0$

(in projection) STARS

$$\frac{1}{N} \sum_{i=1}^{N} v_i^2 = \langle v^2 \rangle = 3\langle v_r^2 \rangle = 3\sigma_r^2$$

DISPERSION

$$\Rightarrow -3m\sigma_r^2 \approx -\frac{3}{5}\frac{GM^2}{NR} = U_g$$

(POTENTIAL OF SPHERICAL DIST.)

$$M = N \cdot m \qquad \Rightarrow M_{virial} \approx \frac{5R\sigma_r^2}{G}$$

OBSERVATION OF M87
(ELLIPTICAL)

HST : (SMALL) DISK OF MATERIAL:
$V \sim 550 \frac{km}{s}$

$\Rightarrow 3.2 \pm 0.9 \times 10^9 M_\odot$ BLACK HOLE

IN GENERAL

$$M_{BH} = \alpha \left(\frac{\sigma}{\sigma_0}\right)^\beta$$

$\alpha = (1.66 \pm 0.24) \times 10^8 M_\odot$

$\beta = 4.86 \pm 0.43$

$\sigma_0 = 200 \frac{km}{s}$

σ: VELOCITY DISPERSION OF STARS IN GALAXY

↑ SLIDE 19

(MUCH FURTHER OUT THAN σ_r BEFORE)

SPIRAL STRUCTURE

- GRAND DESIGN SPIRALS (10%)
  - TWO VERY SYMMETRIC, WELL DEFINED ARMS

  SLIDE M51 (WHIRLPOOL GALAXY)

- FLOCCULENT SPIRALS (30%)

  SLIDE NGC 2841

  NO WELL DEFINED ARMS

- 60% MULTIPLE ARMS

TRAILING ARMS

⇑

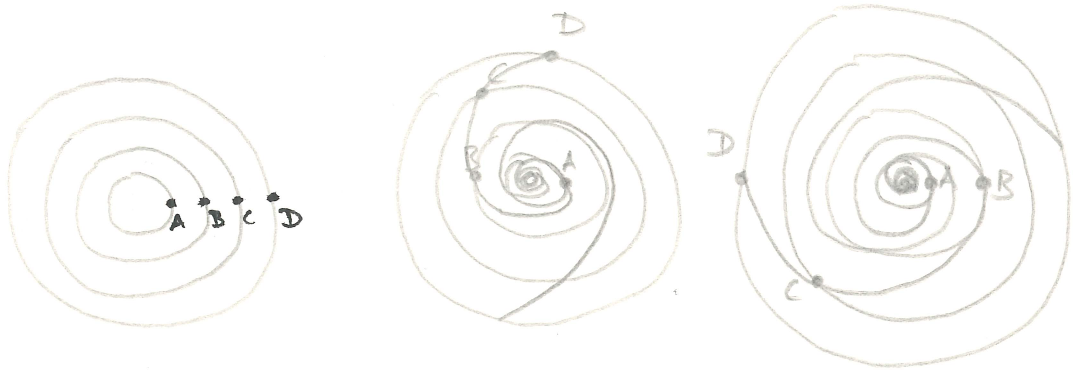
MAJORITY OF CASES

LEADING ARMS

- NGC 4622 : TWO ARMS GO ONE WAY
  ANOTHER ARM OTHER WAY

WINDING PROBLEM

IF SPIRAL CONSISTS OF MATERIAL ARMS (OF STARS AND GAS)



- STARS ALONG LINE (A, B, C, D)
- OUTER STARS REQUIRE MORE TIME TO COMPLETE ORBIT
  - → TRAILING SPIRAL ARMS
- BUT ARMS WIND UP

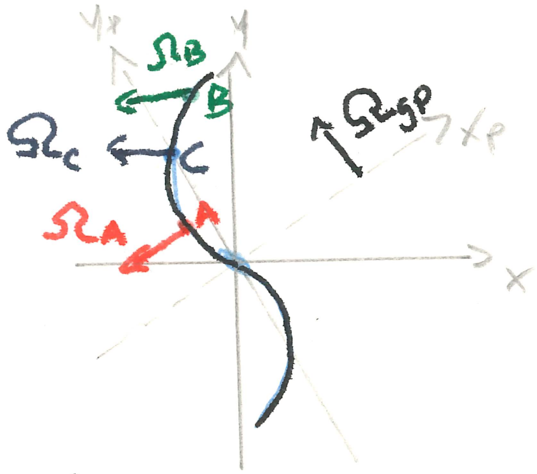
⇒ SPIRAL ARMS ARE <u>NOT</u> MATERIAL ARMS

LIN-SHU DENSITY WAVE THEORY

SPIRAL STRUCTURE: QUASI-STATIC DENSITY WAVES

REGIONS IN GALACTIC DISK WHERE MASS DENSITY GREATER THAN AVERAGE (~10-20%)

- STARS, DUST, GAS CLOUDS <u>MOVE</u> THROUGH DENSITY WAVES (LIKE CARS THROUGH TRAFFIC JAMS)



TRAILING SPIRAL

INERTIAL REFERENC FRAME:

GLOBAL ANGULAR PATTERN SPEED: Ω_{gp}

(NEAR CENTRE)

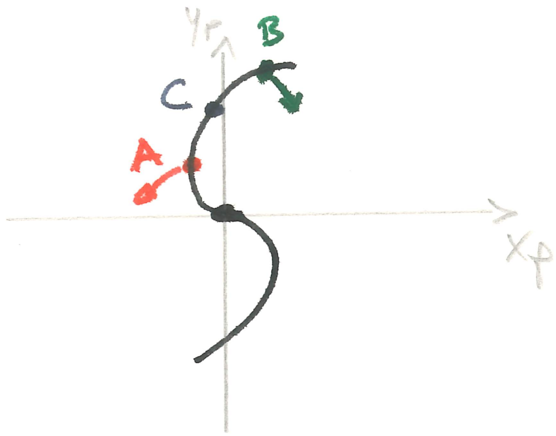
A: $\Omega_A > \Omega_{gp}$

B: $\Omega_B < \Omega_{gp}$

C: $\Omega_C = \Omega_{gp}$ (FAR FROM CENTRE)

CO-ROTATION

FRAME ROTATING WITH GLOBAL PATTERN S':



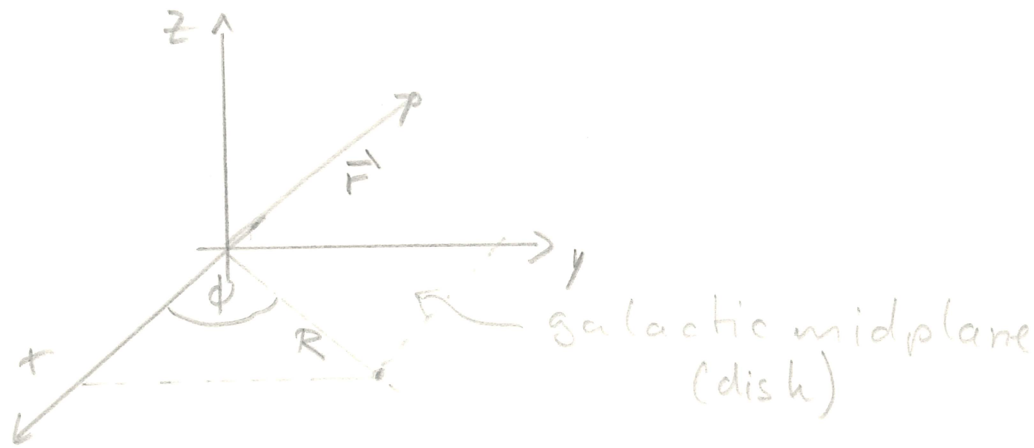
$R < R_c$ STARS PASS THROUGH PATTERN IN ONE DIRECTION

$R > R_c$ STARS PASS THROUGH IN OTHER DIRECTION

SMALL AMPLITUDE ORBITAL PERTURBATIONS

- BASIC UNDERSTANDING OF SPIRAL STRUCTURE

• MOTION OF STARS IN AXISYMMETRIC GRAVITATIONAL FIELD
(DENSITY WAVE NEGLIGIBLE)
NOT ALWAYS VALID



$$\vec{r} = R\,\hat{e}_R + z\,\hat{e}_z$$

$(\hat{e}_R, \hat{e}_\phi, \hat{e}_z)$: cylindrical coordinates

$$x = R\cos\phi$$
$$y = R\sin\phi$$
$$z = z$$

$$\hat{e}_R = \hat{\imath}\cos\phi + \hat{\jmath}\sin\phi$$
$$\hat{e}_\phi = -\hat{\imath}\sin\phi + \hat{\jmath}\cos\phi$$
$$\hat{e}_z = \hat{k}$$

STAR OF MASS M:

$$M\,\frac{d^2\vec{r}^2}{dt^2} = \vec{F}_g(R,z)$$

ϕ NOT HERE BECAUSE OF AXI-SYMMETRY

$$M\,\ddot{\vec{r}} = -\vec{\nabla}u(R,z)$$

$$= -\frac{\partial u}{\partial R}\hat{e}_R - \cancel{\frac{1}{R}\frac{\partial u}{\partial \phi}}\hat{e}_\phi - \frac{\partial u}{\partial z}\hat{e}_z \qquad (*)$$

DEFINE $\Phi = \frac{u}{M}$

$$\ddot{\vec{r}} = -\frac{\partial \Phi}{\partial R}\hat{e}_R - \frac{\partial \Phi}{\partial z}\hat{e}_z$$

$$\ddot{\vec{r}} = (\ddot{R} - R\dot{\phi}^2)\hat{e}_R + \frac{1}{R}\frac{\partial(R^2\dot{\phi})}{\partial t}\hat{e}_\phi + \ddot{z}\hat{e}_z$$

TIME DERIVATIVE IN cylindrical coordinates ($\hat{e}_R, \hat{e}_\phi$ time dependent)

COMPARE TO $(*)$:

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R}$$

$$\frac{1}{R}\frac{\partial(R^2\dot{\phi})}{\partial t} = 0 \qquad (\Delta)$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}$$

Z-COMPONENT OF ANGULAR MOMENTUM:

$$J_z = \frac{L_z}{M} = R^2\dot{\phi} = \text{const.} \qquad (\text{FROM } \Delta)$$

$$\Rightarrow \quad \dot{\phi} = \frac{J_z}{R^2}$$

$$\Rightarrow \quad R\dot{\phi}^2 = \frac{J_z^2}{R^3}$$

$$\Rightarrow \quad \ddot{R} = -\frac{\partial \Phi}{\partial R} + \frac{J_z^2}{R^3}$$

DEFINE EFFECTIVE GRAVITATIONAL POTENTIAL

$$\Phi_{eff}(R,z) \equiv \Phi(R,z) + \frac{J_z^2}{2R^2}$$

$$\Rightarrow \quad \ddot{R} = -\frac{\partial \Phi_{eff}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{eff}}{\partial z}$$

FIND MINIMA OF POTENTIAL:

$$\frac{\partial \Phi_{eff}}{\partial R} = 0$$

$$\frac{\partial \Phi_{eff}}{\partial z} = 0$$

AT $z = 0$
BECAUSE OF
SYMMETRY ABOUT
MIDPLANE

SINCE $\Phi < 0$

AND $\Phi \to 0$ FOR $z \to \infty$

$\Rightarrow$ $z = 0$: MINIMUM

$$\frac{\partial \Phi_{eff}}{\partial R} = \frac{\partial \Phi}{\partial R} - \frac{J_z^2}{R^3} \overset{!}{=} 0 \qquad (\text{MIN.})$$

FOR SOME RADIUS R_m AT $z=0$

$$\left.\frac{\partial \Phi}{\partial R}\right|_{(R_m,0)} = \frac{J_z^2(R_m,0)}{R_m^3} \qquad (**)$$

$$(\vec{L} = \vec{r} \times \vec{p})$$

WITH $J_z = R v_\phi$

$$\frac{J_z^2}{R_m^3} = \left.\frac{v_\phi^2}{R}\right|_{R_m}$$

HENCE $(**)$ IS FOR PERFECT CIRCULAR MOTION

$$F_R(R_m) = -\left.\frac{M v_\phi^2}{R}\right|_{R_m}$$

$\Rightarrow$ MINIMUM FOR Φ_{eff} FOR STAR IN PERFECT CIRCULAR ORBIT IN MIDPLANE

EXPAND AROUND MINIMUM $(R_m, 0)$

$$\xi = R - R_m$$

$$\Rightarrow \Phi_{eff}(R,z) = \Phi_{eff,m} + \cancel{\left.\frac{\partial \Phi_{eff}}{\partial R}\right|_m \xi} + \cancel{\left.\frac{\partial \Phi_{eff}}{\partial z}\right|_m z}$$

$$+ \frac{1}{2}\left.\frac{\partial^2 \Phi_{eff}}{\partial R^2}\right|_m \xi^2 + \cancel{\frac{1}{2}\left.\frac{\partial^2 \Phi_{eff}}{\partial R\,\partial z}\right|_m \xi z}$$

$$+ \frac{1}{2}\frac{\partial^2 \Phi_{eff}}{\partial z^2} z^2 + \dots$$

symmetry ($\pm z$) about $z=0$

DEFINE:

$$\kappa^2 \equiv \left.\frac{\partial^2 \Phi_{eff}}{\partial R^2}\right|_m$$

$$\nu^2 = \left.\frac{\partial^2 \Phi_{eff}}{\partial z^2}\right|_m$$

$$\Rightarrow \Phi_{eff}(R,z) \approx \Phi_{eff,m} + \frac{1}{2}\kappa^2 \varrho^2 + \frac{1}{2}\nu^2 z^2$$

NOTE: $\ddot{\varrho} = \ddot{R}$

$$\Rightarrow \left.\begin{aligned} \ddot{\varrho} &\approx -\kappa^2 \varrho \\ \ddot{z} &\approx -\nu^2 z \end{aligned}\right\} \text{ SIMPLE HARMONIC MOTION}$$

$$\Rightarrow \boxed{\varrho(t) = R(t) - R_m = A_R \sin \kappa t}$$

(choose phase AT t = 0)

κ: EPICYCLE FREQUENCY

$$\boxed{z(t) = A_z \sin(\nu t + \zeta)}$$

ν: VERTICAL OSCILLATION FREQUENCY

ζ: PHASE SHIFT BETWEEN ϱ AND z

FOR COMPLETE DESCRIPTION OF MOTION

$$\dot{\phi} = \frac{V_\phi}{R(t)} = \frac{J_z}{[R(t)]^2}$$

WITH $R(t) = R_m + \varrho(t) = R_m \left[1 + \frac{\varrho(t)}{R_m}\right]$

ASSUME $\varrho(t) \ll R_m$

$$\Rightarrow \dot{\phi} \approx \frac{J_z}{R_m^2}\left(1 - 2\frac{\varrho(t)}{R_m}\right)$$

$\Rightarrow$ USE $\varrho(t)$ AND INTEGRATE

$$\phi(t) = \phi_0 + \frac{J_z}{R_m^2} + \frac{2J_z}{\varkappa R_m^3} A_R \cos \varkappa t$$

CIRCULAR ORBIT

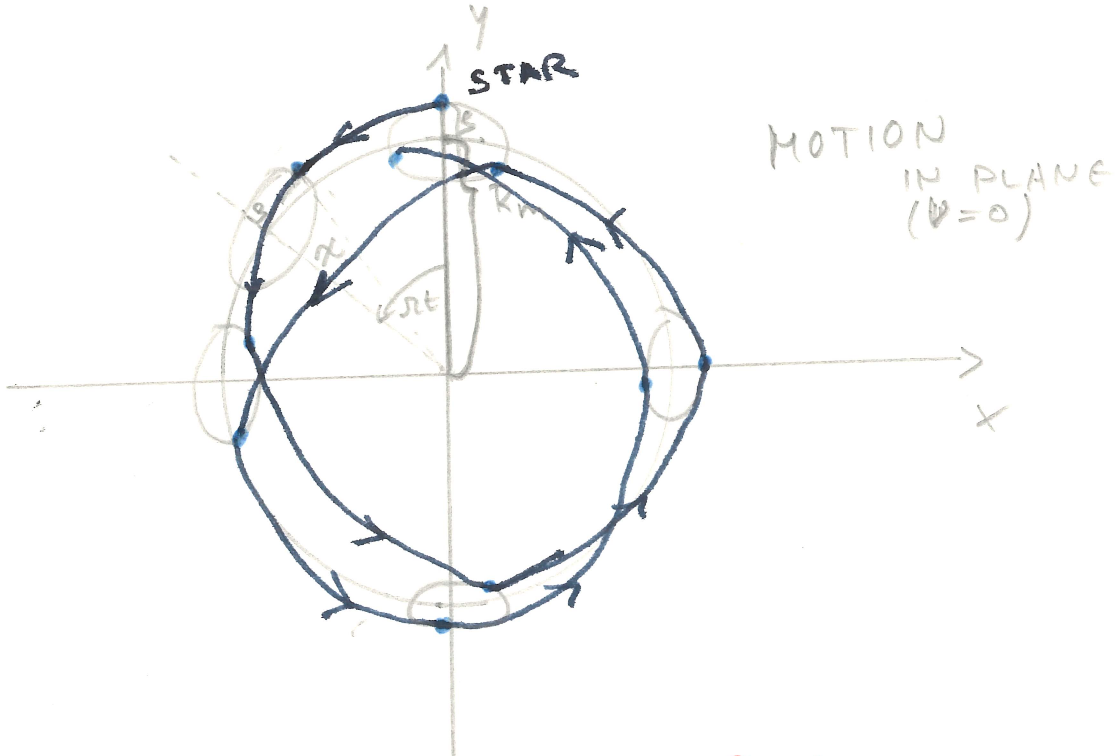
$$= \phi_0 + \Omega t + \frac{2\Omega}{\varkappa R_m} A_R \cos \varkappa t$$

WITH $\Omega \equiv \frac{J_z}{R_m^2}$: ANGULAR SPEED

DEFINE DIFFERENCE IN AZIMUTHAL POSITION FROM EQUILIBRIUM

$$x(t) = \left[\phi(t) - (\phi_0 + \Omega t)\right] R_m$$

$$\boxed{x(t) = \frac{2\Omega}{\varkappa} A_R \cos \varkappa t}$$



ROSETTE PATTERN