LECTURE VII

THE LARGE-SCALE DISTRIBUTION OF GALAXIES; GROUPS AND CLUSTERS

- GALAXIES ARE NOT UNIFORMLY DISTRIBUTED IN SPACE : GROUPS, CLUSTERS, FILAMENTS, SHEETS, SUPER CLUSTERS

SLIDE 68 (59, 60)

RELATION BETWEEN DISTANCE D AND REDSHIFT (VELOCITY V) DUE TO EXPANSION (LOCALLY)

HUBBLE - LAW: $V = H_0 \cdot D$

$$\left(H_0 \approx 70 \frac{km}{s Mpc} \right)$$

~~SLIDE 69~~

TYPICAL SCALES OF LARGE-SCALE STRUCTURE

GALAXIES: $\sim 10\,kpc$

GROUPS & CLUSTERS: $\sim (0.3 \ldots 5)\,Mpc$

SUPER CLUSTERS: $\sim 50\,Mpc$

LARGEST OVERDENSE STRUCTURES NEAR EQUILIBRIUM : LARGE GALAXY CLUSTERS (COMA)

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TWO-POINT CORRELATION FUNCTION OF GALAXIES

MORE DETAILS IN SPECIAL LECTURE

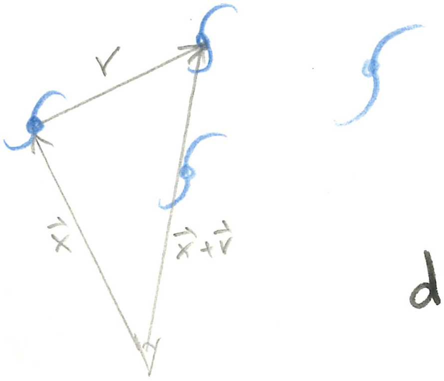
TWO-POINT CORRELATION FUNCTION $\xi(r)$

PROBABILITY FIND AN EXCESS OF PAIRS OF GALAXIES AT DISTANCE r

$$dP_{pair} = n_o^2 \left(1 + \xi(r)\right) dV_1 \, dV_2$$

n_o : MEAN DENSITY OF GALAXIES

dV_1, dV_2 : VOLUME ELEMENT AROUND TWO GALAXIES



RELATIVE OVERDENSITY:

$$\Delta(x) = \frac{\delta n}{n_o}$$

$$dP_{pair} = n(\vec{x}) dV_1 \, n(\vec{x}+\vec{r}) dV_2$$

$$= n_o \left(1 + \Delta(\vec{x})\right)\left(1 + \Delta(\vec{x}+\vec{r})\right) dV_1 dV_2$$

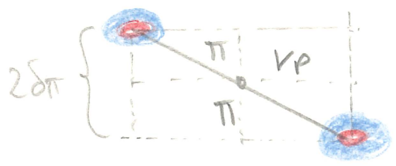
$$\langle dP_{pair} \rangle = n_o^2 \left(1 + \langle \Delta(\vec{x}) \Delta(\vec{x}+\vec{r}) \rangle\right) dV_1 \, dV_2$$

$$\xi(r) = \langle \Delta(\vec{x}) \, \Delta(\vec{x}+\vec{r}) \rangle$$

- OBSERVATIONALLY OVER ALL GALAXY TYPES:

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$ CORRELATION FUNCTION

PROJECTED CORRELATION FUNCTION



π : $\parallel$ TO LINE OF SIGHT

r_p : $\perp$ 〃

↑

r_p : PROJECTED DISTANCE BETWEEN PAIRS OF GALAXIES

∀

$$w_p(r_p) = \int_{-\delta\pi}^{+\delta\pi} \xi(r_p, \pi)\, d\pi$$

PROJECTION :
$$W_p(r_p) = 2\int_0^\infty dy\, \xi\left[\left(r_p^2+y^2\right)^{1/2}\right]$$

$r^2 = r_p^2 + y^2$

$$= 2\int_{r_p}^\infty r\,dr\, \xi(r)\left(r^2-r_p^2\right)^{-1/2}$$

$$W_p(r_p) = r_p \left(\frac{r_p}{r_0}\right)^{-\gamma} \Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{\gamma-1}{2}\right)\Big/\Gamma\left(\frac{\gamma}{2}\right)$$

SLIDE - 70 (61)

$\Gamma(n) = n!$

$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$

APM GALAXY SURVEY (1996):

$\gamma = 1.8$ $r_0 \approx 5h^{-1}\,\mathrm{Mpc}$

$\left(h = \frac{H_0}{100 \frac{km}{s\,Mpc}}\right)$

VALID: $\sim 200\,\mathrm{kpc} < r < 10\,\mathrm{Mpc}$

- ELLIPTICAL, S0 GALAXIES: (red)
  PREFER HIGH DENSITY REGIONS

- SPIRALS, IRREGULARS : LOWER DENSITY REGION (blue)

- ALL TYPES FOUND IN FILAMENTS, GROOPS AND GALAXY CLUSTERS

- DIFFERENT GALAXIES (RED, BLUE) · SLIDE-71 (62)
  ↑ ELLIPTICALS ↑ SPIRALS

  CLUSTER DIFFERENT

- DIFFERENT AMPLITUDE AT DIFFERENT REDSHIFTS → GROWTH

- LARGER SAMPLES (SDSS):
  CORRELATION FUNCTION IS NOT POWER LAW → SLIDE-72

LOCAL GROUP

- MILKY WAY : BELONGS TO LOOSE COLLECTION OF GALAXIES
  SLIDE - 73 (64)
  LOCAL GROUP

- BRIGHTEST MEMBERS :

  - ANDROMEDA (M31)
  - MILKY WAY
  - M33

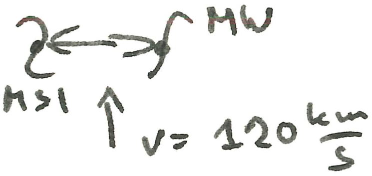
  SPIRAL

  (APART FROM M32, NO ELLIPTICALS)

  - Irr (Large MC , SMC) ; DWARF SPHEROIDALS

- MEMBERS : 40+

- GRAVITATIONALLY BOUND
  (M31 , MILKY WAY)



- TOTAL MASS : $\approx$ few × $10^{12}\, M_\odot$

GALAXY CLUSTERS

OPTICAL PROPERTIES

- RICHNESS : NUMBER OF CLUSTER GALAXIES
  30 − 300
- RADIUS : SURFACE DENSITY DROPS TO 1% OF THE CORE DENSITY
  1 − 2 Mpc
- RADIAL VELOCITY DISPERSION: 400 − 1400 $\frac{km}{s}$
- MASS $(r < 1.5\,\mathrm{Mpc})$: $10^{14} - 10^{15}\ M_\odot$
- OPTICAL B-BAND LUMINOSITY: $(r < 1.5\,\mathrm{Mpc})$
  $10^{11} - 10^{13}\ L_\odot$
- MASS-TO-LIGHT RATIO : $\approx 300\ M_\odot / L_\odot$
- CLUSTER NUMBER DENSITY: $10^{-6} - 10^{-5}\ \mathrm{Mpc}^{-3}$
- CLUSTER CORRELATION SCALE: 22 ± 4 Mpc
- FRACTION OF GALAXIES WITH $L > L_*$ IN CLUSTERS:
  ~ 5%

SLIDE - 74 (65)

X-RAY GAS IN GALAXY CLUSTERS

- BARYONIC GAS COMPRESSED IN CLUSTER POTENTIAL WELL
  → SHOCK HEATING TO X-RAY TEMPERATURE

- SPECTRA: BREMSSTRAHLUNG WITH TEMPERATURE 10^8K GAS

thermal-bremsstrahlung $\quad \varepsilon_\nu = \frac{dL}{dV d\nu}$

free-free EMISSIVITY:

$$\varepsilon(\nu) = \frac{32\pi Z^2 n_e n_i}{3 m_e c^3} \sqrt{\frac{2\pi}{3 k m_e T}} \exp\left(-\frac{h\nu}{kT}\right) g_{ff}(T,\nu)$$

g_{ff} : GAUNT FACTOR (RADIATIVE CORRECTIONS)

INTEGRATING OVER FREQUENCY:

$$\varepsilon = 2.4 \times 10^{-27}\, T^{-1/2} n_e^2 \left[\frac{\text{erg}}{\text{cm}^3\, \text{s}}\right]$$

THE COOLING TIME OF THE PLASMA IS:

$$t_{cool} = \frac{3 n_e k T}{n_e^2} = \frac{10^{11}}{n_e} T^{1/2} [s]$$

X-RAY SURFACE BRIGHTNESS:

$$S(R) = S(0)\left[1 + \frac{R^2}{r_{core}^2}\right]^{-3\beta + 1/2}$$

(β-MODEL)

X-RAY CLUSTER PROPERTIES

FOR RICH CLUSTERS

- TEMPERATURE : 2 - 14 keV OR $2 \times 10^6 - 10^8$ K
- LUMINOSITY : $10^{42.5} - 10^{45}$ erg/s
- CORE RADIUS : 0.1 - 0.2 Mpc
- CENTRAL ELECTRON DENSITY : $n_e \simeq 10^{-3} \text{cm}^{-3}$
- GASS MASS : $M_{gas} = 10^{13} - 10^{14} M_\odot$
- Fe ABUNDANCE : $\approx 1/3$ SOLAR

CENTER OF COMA CLUSTER : $L = 10^{44}$ erg/s

$\bar{n}_e = 10^{-3} \text{cm}^{-3}$

SLIDE 75

$\bar{\tau}_{cool} \simeq 10^{10}$ yrs

$M_{gas} \simeq 10^{13} M_\odot$

MASS DETERMINATION METHODS

FOR GALAX~~IES~~Y CLUSTERS

FROM DYNAMICS : VIRIAL THEOREM

$$M = \frac{r_G \langle v^2 \rangle}{G}$$

$$r_G = \frac{\sum_i m_i}{\sum_j \sum_{i \neq j} \frac{m_i \cancel{m_j}}{r_{ij}}} \quad \text{AND} \quad \langle v^2 \rangle = \frac{\sum_i m_i (v_i - \bar{v})^2}{\sum_i m_i}$$

FROM X-RAYS:

HYDROSTATIC EQUILIBRIUM:

$$\frac{1}{\rho_g}\frac{dP}{dR} = -\frac{GM(<r)}{r^2}$$

AS FOR ELLIPTICAL GALAXIES:

$$M(<r) = -\frac{kT(r)r}{G\mu m}\left[\underbrace{\frac{d\ln\rho_g(r)}{dr}}_{\uparrow\ \text{density profile}} + \overbrace{\frac{d\ln T(r)}{dr}}^{\text{Temperature profile}\ \downarrow}\right]$$

MAIN COMPONENTS OF GALAXY CLUSTERS

- DARK MATTER (80% - 87% OF TOTAL MASS)
  - DETERMINES GRAVITATIONAL POTENTIAL AND DYNAMICS
  - BACKBONE OF SYSTEM
  - DETAILED PREDICTIONS ONLY FROM SIMULATIONS
  - ONLY INDIRECT OBSERVATIONS POSSIBLE
- INTRA CLUSTER MEDIUM (11 - 14% OF THE TOTAL MASS)
  - DOMINATES BARYONIC MASS
  - ~~COMPLE~~ VERY THIN AND HOT PLASMA $(\sim 10^{8}\,°K)$
  - EMITS X-RAYS

○ GALAXIES

· (2-5% OF THE TOTAL MASS)

100s - 1000s OF GALAXIES

○ CORRELATIONS BETWEEN COMPONENTS

- CENTRAL GALAXY DENSITY HIGHER FOR HIGHER L_X
- FRACTION OF SPIRALS LOWER FOR HIGHER L_X
- TEMPERATURE T ~ L_X ABOUT 10^8 K
- RATIO OF GAS-MASS TO GALAXY-MASS INCREASES WITH T UP TO 5 OR MORE

○ DOMINANT COMPONENT: DARK MATTER

- ○ GALACTIC DYNAMICS
- ○ x-ray gas (hydro equilib.)
- ○ GRAVITATIONAL LENSING

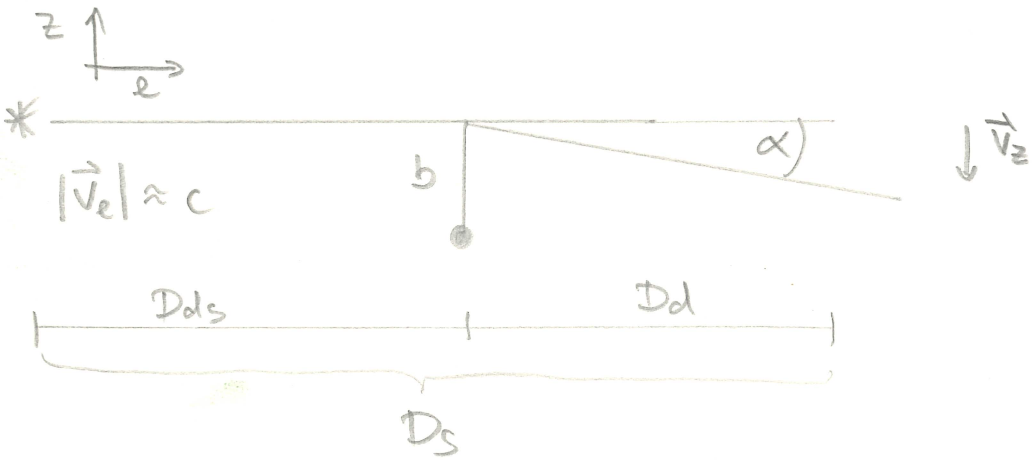
SLIDE - 76

MASSES OF GALAXY CLUSTERS

FROM GRAVITATIONAL LENSING

SLIDE-78

- GENERAL RELATIVITY: LIGHT RAYS ARE DEFLECTED BY MASS $(v \ll c,\ \phi \ll c^2)$ (EINSTEIN: SUN 1,7")



- $b \ll Dd$, $b \ll Dds$
- NEAR LENS LIGHT DEFLECTION

acceleration in z-direction

NEWTONIAN APPROXIMATION: $\alpha = \frac{V_z}{c} = \frac{1}{c}\int \frac{d\phi}{dz}\,dt$

$$= \frac{1}{c^2}\int \frac{d\phi}{dz}\,dl$$

- GRAVITATIONAL LENSES ARE ACHROMATIC !

- RELATIVISTIC RESULT

$$\vec{\alpha} = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi\, dl$$

POINT MASS:

$$\phi(l,z) = -\frac{GM}{(l^2+z^2)^{1/2}}$$

$$\Rightarrow \frac{d\phi}{dz} = \frac{GM}{(l^2+z^2)^{3/2}} \qquad (= \vec{\nabla}_\perp \phi)$$

$$\Rightarrow \alpha = \frac{4GMz}{c^2}\left[\frac{l}{z^2(l^2+z^2)^{1/2}}\right]_0^8$$

$$\alpha = \frac{4GM}{c^2 b} = \frac{2R_s}{b}$$

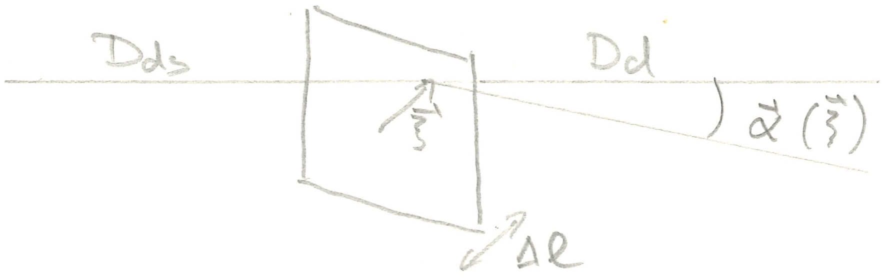
(R_s: Schwarzschild Radius)

SUN: $M_\odot \simeq 2\cdot 10^{33}\,g \rightarrow R_s \simeq 3\,km$

$$\Rightarrow \alpha_{\odot,R_\odot} \simeq 1.7''$$

FOR ARBITRARY MASS DISTRIBUTION:

EXTEND OF MASS DISTRIBUTION SMALL COMPARED TO DISTANCE: $\Delta l \ll D_{ds}$; $\Delta l \ll D_d$



TREATED LIKE INFINETLY THIN MASS SHEET $\Rightarrow$ SURFACE MASS DENSITY:

$$\Sigma(\vec{\xi}) = \int_{\Delta l} \rho(\vec{\xi}, l)\, dl$$

DEFLECTION BY MASS ELEMENTS OF LIGHT RAY PASSING AT $\vec{\xi}$

$$dm = \Sigma(\vec{\xi}')\, d^2\vec{\xi}' :$$

$$d\vec{\alpha} = \frac{4\,G\,dm}{c^2 |\vec{\xi}' - \vec{\xi}|}$$

INTEGRATING ALL DEFLECTIONS DIRECTIONS $\dfrac{(\vec{\xi} - \vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|}$

$$\Rightarrow \vec{\alpha}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\, \Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2}\, d^2\xi'$$

FOR SPHERICAL MASS DISTRIBUTION:

$$\alpha(\xi) = \frac{4GM(<\xi)}{c^2\, \xi}$$

WITH $M(<\xi) = 2\pi \int_0^{\xi} \Sigma(\xi')\, \xi'\, d\xi'$