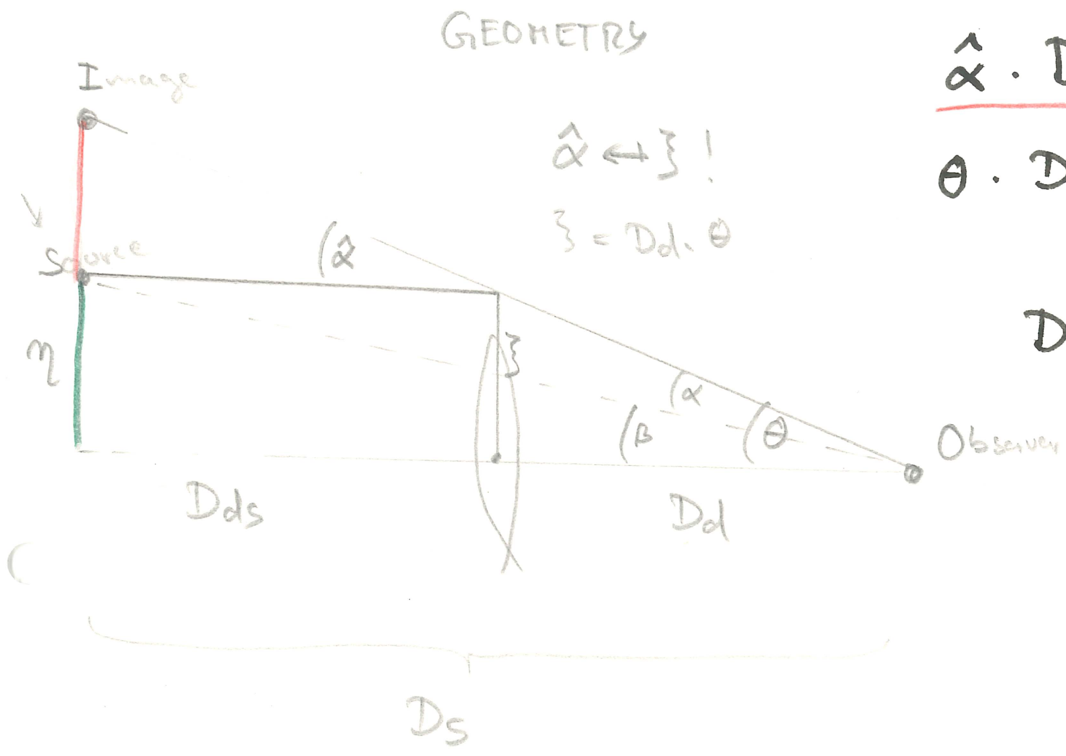
LENS EQUATION



$$\hat{\alpha} \cdot D_{ds} = \alpha \cdot D_s$$

$$\theta \cdot D_s = \beta \cdot D_s + \hat{\alpha}\, D_{ds}$$

D: ANGULAR DIAMETER DISTANCE (SEE LATER)

$D_{ds} \neq D_s - D_d$!

$$\Rightarrow \quad \beta = \theta - \alpha = \theta - \frac{D_{ds}}{D_s}\hat{\alpha}$$

LENS EQUATION

(RELATES REAL POSITION (ANGLE) OF SOURCE (WITHOUT LENS) WITH POSITION OF LENSED IMAGE)

EINSTEIN RADIUS

~~SLIDE 69~~

- CIRCULAR SYMMETRIC LENS
- SOURCE ON OPTICAL AXIS

$\Rightarrow \quad \beta = 0 \quad \Leftrightarrow \quad \theta = \alpha$

$$= \frac{D_{ds}}{D_s} \hat{\alpha}$$

$$= \frac{D_{ds}}{D_s} \frac{4G\, M(<\xi)}{c^2\, \xi}$$

$$= \frac{D_{ds}}{D_s} \cdot \frac{4\pi G}{c^2} \frac{M(<\xi)}{\pi \xi^2} \xi$$

$$= \frac{D_{ds}}{D_s} \cdot \frac{4\pi G}{c^2} \frac{M(<\xi)}{\pi \xi^2} \cdot \underbrace{D_d \cdot \theta}$$

see sketch $\xi = D_d \cdot \theta$

$$= \frac{D_{ds}}{D_s} \cdot \Sigma_{cr} \cdot D_d \cdot \theta \cdot \frac{4\pi G}{c^2}$$

$\Rightarrow$ CRITICAL SURFACE DENSITY

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \cdot \frac{D_s}{D_{ds} D_d} = 0.3 \frac{g}{cm^2} \frac{D_s}{D_{ds}} \frac{1\,Gpc}{D_d}$$

$\Rightarrow$ IF RING IS OBSERVED AND DISTANCES ARE KNOWN $\Rightarrow$ MASS !

SLIDE 69

RADIUS : $\xi = D_d \cdot \theta$

EINSTEIN RADIUS:

$$\Sigma_{cr} = \frac{M(<\xi_E)}{\pi \xi_E^2} = \frac{c^2}{4\pi G} \frac{D_s}{D_{ds} D_d} \qquad (\xi_E = D_d \cdot \theta_E)$$

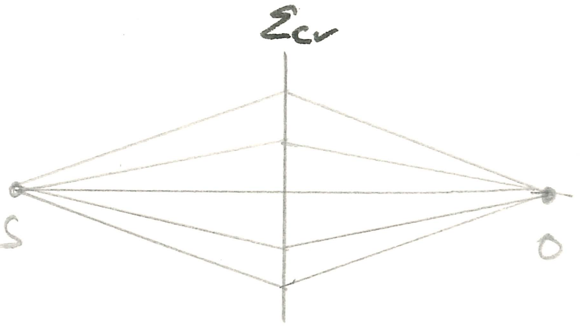
$$\Rightarrow \quad \theta_E^2 = \frac{D_{ds}}{D_s D_d} \frac{4G}{c^2} M_{<\theta_E}$$

IF SURFACE MASS DENSITY HAS Σ_{cr}

AND IS CONSTANT IN ξ

$\Rightarrow$ IDEAL CONVEX LENS

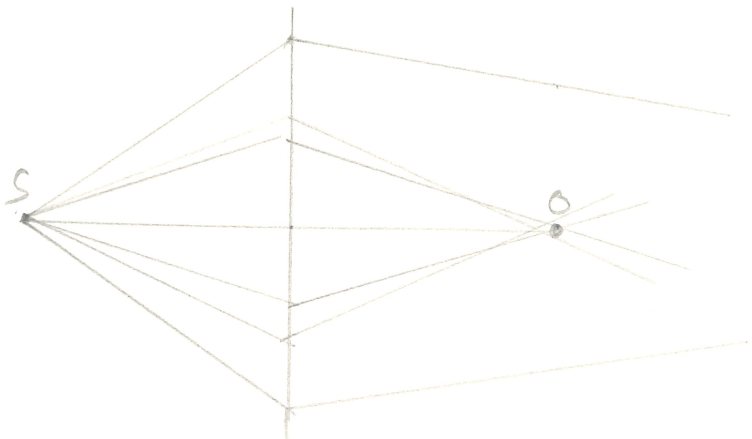
$\Sigma = \text{const.} = \Sigma_{cr}$



BUT: TYPICALLY Σ DECREASES WITH ξ

$\Rightarrow$ ONLY CERTAIN RADIUS FULLFILLS CONDITION

$\Rightarrow$ CIRCULAR IMAGE:



RING

BUT: NOT SPHERICAL SYMMETRIC, ELLIPTICAL

$\Rightarrow$ ARCS (PART OF RING)

EXAMPLES EINSTEIN ANGLES θ_E

1. GALAXY CLUSTERS

TYPICAL MASS : $M \simeq 10^{14} M_\odot$

TYPICAL DISTANCES : ≈ 1 Gpc

LEADS TO : $\theta_E \simeq 10'' \left(\frac{M}{10^{13} M_\odot}\right)^{1/2} \left(\frac{D}{\text{Gpc}}\right)^{-1/2}$

$$D = \frac{D_s D_d}{D_{ds}}$$

2. STARS IN MILKY WAY

$$\theta_E \simeq 0.001'' \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D}{10\,\text{kpc}}\right)^{-1/2}$$

UNOBSERVABLE ; BUT AMPLIFICATION

$\Downarrow$

MICROLENSING

$\Downarrow$

DARK MATTER FROM COMPACT OBJECTS

SLIDE 78-79 (70)

THE HOMOGENOUS UNIVERSE

- BIG BANG SCENARIO

BASIC OBSERVATIONS; ASSUMPTIONS

1. GALAXIES MOVE AWAY FROM EACHOTHER; WITH VELOCITY INCREASING WITH DISTANCE SLIDE - 80 (71)

2. ON LARGE SCALE UNIVERSE IS ISOTROPIC (DISTRIBUTION OF FAINT GALAXIES) RADIO SOURCES, MICROWAVE BACK GROUND) SLIDE-69, SLIDE-81, SLIDE-82 71, 73

3. OUR LOCATION AND OBSERVATIONS ARE NOT UNIQUE BUT TYPICAL

   COSMOLOGICAL PRINCIPLE

4. GRAVITATION AND HENCE DYNAMICS OF THE UNIVERSE IS DESCRIBED BY EINSTEIN'S GENERAL RELATIVITY!

GENERAL RELATIVITY

EINSTEIN EQUATIONS

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(SIGN CONVENTION IN METRIC!)

"DISTANCES" IN SPACE TIME: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$R_{\mu\nu}$: Ricci-Tensor $(R_{\mu\nu}(g_{\mu\nu}))$ $\Longleftrightarrow$ SPACE-TIME CURVATURE

$g_{\mu\nu}$: METRIC TENSOR $\Longleftrightarrow$ SPACE-TIME CURVATURE

R : Ricci-SCALAR $\Longleftrightarrow$ SPACE-TIME CURVATURE

G : NEWTON'S CONSTANT

$T_{\mu\nu}$: ENERGY-MOMENTUM TENSOR $\Longleftrightarrow$ MASS, ENERGY

Λ : COSMOLOGICAL CONSTANT

CONNECT ENERGY (DENSITY) TO GEOMETRICAL PROPERTIES (CURVATURE)

THE ROBERTSON-WALKER METRIC

HOMOGENEOUS, ISOTROPIC 3-SPACE

$$ds^2 = g^{RW}_{\mu\nu}\, dx^\mu dx^\nu$$

$$= c^2 dt^2 - a^2(t)\left[dr^2 + K^{-1}\sin^2\left(K^{1/2} r\right)\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)\right]$$

(r, θ, φ): COMOVING SPHERICAL COORDINATES

$a(t)$: COSMIC SCALE FACTOR

K: CURVATURE PARAMETER (CONSTANT)

$$K^{-1/2}\sin\left(K^{1/2} r\right) = \begin{cases} K^{-1/2}\sin\left(K^{1/2} r\right) & (K>0;\ \text{POSITIVE CURVATURE}) \\ (-K)^{-1/2}\sinh\left((-K)^{1/2} r\right) & (K<0;\ \text{NEGATIVE}) \\ r & (K=0\ \text{FLAT}) \end{cases}$$

SLIDE - 83 (71)

CONVENIENT DEFINITION:

$$k = \frac{K}{|K|} \qquad (K \neq 0)$$

$$k = 0 \qquad (K=0)$$

$$R_{c,0}^2 = |K|^{-1}$$

$(k = +1, -1, 0)$ $\qquad R_{c,0}$: ABSOLUT VALUE OF CURVATURE TODAY

$\Rightarrow$

$$* \; ds^2 = c^2 dt^2 - a^2(t)\left[dr^2 + R_{c,0}^2 S_k^2\left(\frac{r}{R_{c,0}}\right)\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)\right]$$

(MOSTLY USE THIS ONE)

WITH
$$S_k\left(\frac{r}{R_{c,0}}\right) = \begin{cases} \sin\left(\frac{r}{R_{c,0}}\right) & k=1 \\ \sinh\left(\frac{r}{R_{c,0}}\right) & k=-1 \\ \frac{r}{R_{c,0}} & k=0 \end{cases}$$

NORMALIZATION: TODAY (t_0)

$$a(t_0) = 1$$

$\Rightarrow$ dr : REAL DISTANCES TODAY

OTHER POPULAR CHOICES: $R(t) = a(t)\cdot R_{c,0}$

$$r' = \frac{r}{R_{c,0}}$$

$$\Rightarrow \; ds^2 = c^2 dt^2 - R(t)^2\left[dr'^2 + S_k^2(r')\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)\right]$$

$$R(t_0) = R_{c,0}$$

OTHER CONVENIENT CHOICE: $S_k(r') = \hat{r}$

$$\Rightarrow \; ds^2 = c^2 dt^2 - R(t)^2\left[\frac{d\hat{r}^2}{1-k\hat{r}^2} + \hat{r}^2 \underbrace{\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right)}_{d\Omega^2}\right]$$