Problem set #1

Problem 1 *Kepler's 3rd law*

Suppose two stars with masses m_1 and m_2 are in circular orbits, with radii r_1 and r_2, about their common center of mass, and that their orbital period is P.

(a) Show that $r_1 = \frac{m_2}{(m_1+m_2)} r$ and $r_2 = \frac{m_1}{(m_1+m_2)} r$, where $r = r_1 + r_2$ is the distance between the two stars.

(b) The speeds of the stars in orbit are constant and given by $v_1 = \Omega r_1$ and $v_2 = \Omega r_2$, where $\Omega = 2\pi/P$ is the angular speed of revolution of either star about their center of mass. Show that Newton's second law, applied to either star, leads to

$$m_1 + m_2 = \frac{\Omega^2 r^3}{G}.$$

This is of course Kepler's third law, in a slightly different form than usual.

Problem 2 *Radial velocity variations caused by planets*

(a) Derive the amplitude of the radial velocity variation of the host star of a transiting extra-solar planet (inclination $i = 90°$) as a function of period and planetary and stellar mass. Use Kepler's third law in the following approximations:

$$a_{\text{star}} \ll a_{\text{planet}}, \quad (1)$$
$$M_{\text{planet}} \ll M_{\text{star}}. \quad (2)$$

(Hint: use the center of mass theorem and the above approximations to derive a_{planet} as a function of M_{star} and p.)

(b) Calculate the amplitude of the Sun's radial velocity variation caused by Earth, Jupiter, and Saturn ($M_{\text{Earth}} = 5.97 \times 10^{24}$ kg, $M_{\text{Jupiter}} = 1.90 \times 10^{27}$ kg, $M_{\text{Saturn}} = 5.68 \times 10^{26}$ kg, $M_{\text{Sun}} = 1.99 \times 10^{30}$ kg, $P_{\text{Jupiter}} = 11.86$ yr, $P_{\text{Saturn}} = 29.46$ yr).

Problem 3 *Transit detection*

What fraction of sunlight is blocked when Earth passes in front of the Sun and how large is the decrease in brightness expressed in magnitudes? Here it shall be assumed that the observer resides far outside the solar system and that the Sun and the planets appear as uniform disks. How large is the effect for Mercury, Jupiter, and Neptune ($R_{\text{Mercury}} = 0.38 R_{\text{Earth}}$, $R_{\text{Earth}} = 0.00915 R_\odot$, $R_{\text{Jupiter}} = 11.2 R_{\text{Earth}}$, $R_{\text{Neptune}} = 3.89 R_{\text{Earth}}$)?

Problem 4 *Transit probability*

Show that the probability p for a suitable orientation of a planet's orbital plane to allow observing a transit is given by the simple formula

$$p = \frac{R_\star}{a},$$

where $R_\star$ is the radius of the star and a is the planet's orbital radius. How large are the probabilities for an alien observer to be in a position allowing the observation of transits of Mercury, Earth, Jupiter, Neptune ($a_{\text{Mercury}} = 0.387 a_{\text{Earth}}$, $a_{\text{Earth}} = 215 R_\odot$, $a_{\text{Jupiter}} = 5.205 a_{\text{Earth}}$, $a_{\text{Neptune}} = 30.14 a_{\text{Earth}}$)?

Problem 5 *Transit duration*

Derive a general expression for the duration of a transit across the center of a star by using Kepler's third law. Assume that the observer is at a very large distance and that the orbits are circular. The formula should give the transit duration t in hours, when the mass $M_\star$ of the star is given in $M_\odot$, the radius $R_\star$ of the star in $R_\odot$, and the planet's orbital radius a_{p} in AU. How long does a central transit of Mercury, Earth, Jupiter, and Neptune last? How long does the transit last relative to the total orbital period of the planet?