Problem set #2

Problem 1 *Planck function*

The Planck function, defined here in terms of a specific intensity *with respect to frequency* (i.e., with units $\mathrm{erg\,cm^{-2}\,sec^{-1}\,Hz^{-1}\,sr^{-1}}$), is given by

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{k_\mathrm{B}T}\right)-1},$$

with Planck's constant h, frequency ν, speed of light c, Boltzmann's constant k_B and temperature T.

(a) Derive the expression for the corresponding energy density, u_ν.

(b) What is the corresponding function B_λ, i.e., with units $\mathrm{erg\,cm^{-2}\,sec^{-1}\,cm^{-1}\,sr^{-1}}$?

(c) Show that Planck curves for different T do not cross each other.

(d) Prove the Stefan-Boltzmann law and *calculate* the Stefan-Boltzmann constant σ_B.

(e) Prove Wien's displacement law for both B_ν and B_λ and calculate the corresponding constants. (Hint: the zeros of

$$3\exp(x_1) - 3 - x_1\exp(x_1) = 0 \qquad \text{and} \qquad 5\exp(x_2) - 5 - x_2\exp(x_2) = 0$$

are $x_1 \approx 2.82144$ and $x_2 \approx 4.96511$, respectively.)

Problem 2 *Flux received from an ideal star*

- What is the flux received on Earth from a star that emits blackbody radiation, has radius R, and is at a distance r from Earth?
- What is the intensity I_ν?
- Why intensity I_ν does not change with the distance from the source, while the flux F_ν does?
- Given a constant (volume) opacity κ_ν around the star, what is the distance at which the original intensity is reduced by a factor 100 (if the stellar radiation dominates)?
- What if instead the emission from the gas around the star is comparable with the background radiation (i.e. $I_\nu^0 = S_\nu^0$)?