Problem set #3

Problem 1 *Gravitational radius of a disk*

Atomic hydrogen in a protoplanetary disk is ionized by the irradiation of the central star. The temperature of the ionized gas is $T = 10\,000$ K.

(a) Calculate the most likely speed $v_{\rm p}$ of protons in the ionized gas.

(b) Find the formula for the escape velocity $v_{\rm s}$ from a distance r from a star of mass M_*. The disk mass is negligible compared to the mass of the star.

(c) Calculate the *gravitational radius* $r_{\rm g}$ in AU where the ionized hydrogen can photo-evaporate from the disk around a Herbig Ae star of mass $M_* = 2M_\odot$.

(d) Do the same for a T Tauri star of mass $M_* = 1M_\odot$.

(e) Based on the above, what do you think will happen during the evolution of the proto-planetary disk? Can this be observed?

Problem 2 *Virial theorem and Jeans mass*

In the lectures, the Jeans mass was inferred using simple approximations. Here we will derive the Jeans mass using the *virial theorem*.

(a) Show that for a self-gravitating spherically-symmetric ideal-gas cloud in (hydrostatic) pressure equilibrium,

$$E_{\rm kin} = -\frac{1}{2}E_{\rm pot}.$$

(*Hint:* in hydrostatic equilibrium $\frac{dP}{dr} = -\frac{GM_{\rm r}\rho}{r^2} = -\rho g$, where $M_{\rm r}$ is the mass within radius r and g is the local acceleration of gravity at radius r.)

(b) For a cloud to be able to contract, what can we then say about the energies involved? What minimum mass must a cloud of given mean density and mean temperature therefore at least have to contract? Compare your result to the one from the lectures and discuss possible differences.

Problem 3 *Cooling and heating*

We have a cloud of neutral atomic hydrogen. The cloud is in thermal equilibrium with the cosmic-ray heating and the radiative cooling via [CII] forbidden line emission at 157 μm.

(a) Calculate the heating rate Γ by the cosmic-ray ionization in $\mathrm{erg\,cm^{-3}\,s^{-1}}$. For each ionization of the neutral hydrogen, the primary electron eventually deposits E_{dp} =5 eV to the gas through elastic collisions. The cosmic-ray ionization rate of hydrogen is $\zeta_{\mathrm{H}} = 10^{-16}\,\mathrm{s^{-1}}$. The gas density is $n_{\mathrm{H}} = 10\ \mathrm{cm^{-3}}$.

(b) Derive a formula for the number of collisions that a single CII ion experiences per second. Use the cross section σ for CII and the gas density n_{H}. Note that typical velocity of hydrogen at the temperature T is $v = \sqrt{3kT/m_{\mathrm{H}}}$. CII is stationary in comparison to the hydrogen.

(c) Calculate the cooling rate Λ by the radiation via [CII] forbidden line in $\mathrm{erg\,cm^{-3}\,s^{-1}}$ at the temperature 10 K. Assume each collision leads to an excitation of [CII] $^2\mathrm{P}_{1/2} \rightarrow {}^2\mathrm{P}_{3/2}$, and each excited [CII] eventually emits a photon at the wavelength 157 μm. Hydrogen is the sole collision partner. Use [CII]/[H] $=3 \times 10^{-4}$, $\sigma = 10^{-16}\,\mathrm{cm^{-2}}$, and n_{H} $=10\,\mathrm{cm^{-3}}$.

(d) Find T so that heating rate Γ balances with cooling rate Λ.