Problem set #6

Problem 1 *Parallaxes*

The European Space Agency satellite Hipparchos has provided us with the best parallax (and hence distance) measurements of stars in our galaxy before the Gaia era. It measured the trigonometric parallax of stars down to a magnitude of ~ 11 with a precision of ~ 0.001 arcsec.

(a) What is the greatest distance to a star that may be measured with this precision in trigonometric parallax? Compare this result to the distance to some of the Milky Way's globular clusters (e.g., 47 Tucanae, M62, etc., see also page 29 in Milky Way I lecture slides).

(b) At this distance, what is the spectral type of the dimmest/smallest/coolest main sequence star which would be visible down to a magnitude limit of $m_V = 11$? (Hint: use the Hertzsprung-Russell diagram from problem set 5.)

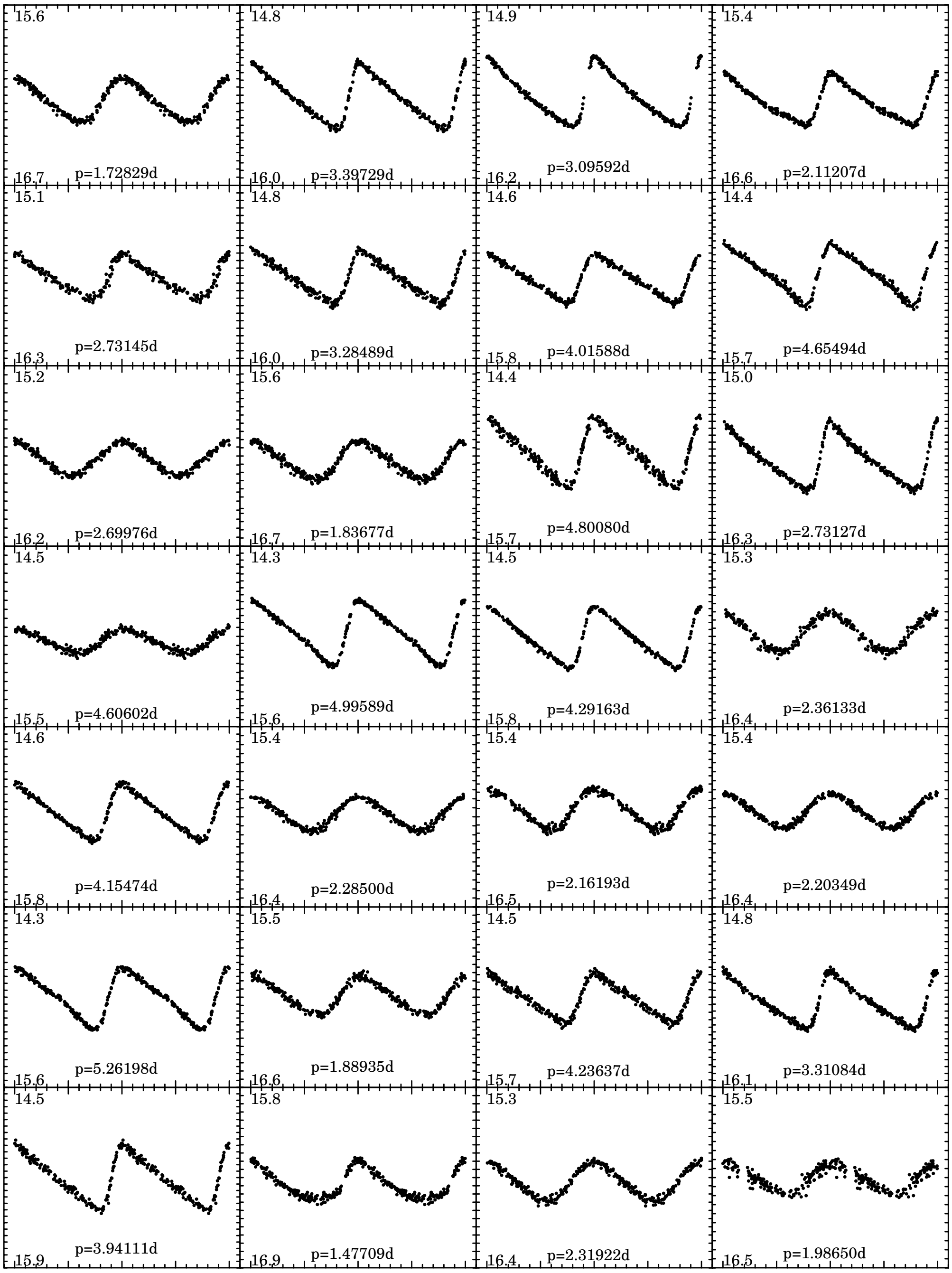
(c) As an Earth satellite, Hipparchos used the same baseline as any other Earth-based observer. Suppose a similar instrument which could measure star positions to the same precision of 0.001 arcsec were placed in orbit around Jupiter; what would be the greatest distance to a star this new spacecraft could measure? (Jupiter is about 5.2AU away from the Sun.) How does this new distance limit compare to the distance to some of the Milky Way's globular clusters?

(d) At what distance from the Sun would the same instrument (now in orbit around the Sun) need to be in order to measure the distance to a star in the Andromeda galaxy (distance 780 kpc) using trigonometric parallax methods? Compare this to the orbit of Neptune (30AU). Note that Neptune takes about 165 years to make a complete orbit around the Sun.

(e) Consider the vastly more sensitive Gaia satellite. Gaia Data Release 2 provides parallax measurement with a precision of $\sim 4 \times 10^{-5}$ arcsec down to a magnitude of 15. What is the greatest distance to a star that can be measured with this precision. What is the spectral type of the faintest star that Gaia may observe at the parallax distance limit of Hipparchos?

Problem 2 *Large Magellanic Cloud distance with period-luminosity relation*

The figure below shows light-curves of Cepheid variable stars, located in the Large Magellanic Cloud. The data was obtained in the OGLE project. Tick marks on the y-axis are spaced 0.05 mag apart. The period in days is also given for each object.
Use the period-luminosity relationship $M_V \approx -2.7 \log_{10}(T/\text{days}) - 1.6$ to calculate a distance for each cepheid.
Calculate the mean distance of LMC and the error of your distance estimate (in pc). Convert this back to an estimate for the distance modulus for LMC.



Problem 3 *Kepler's third law and the Oort constants*

(a) Beginning with Kepler's third law, derive a expression for the orbital velocity, $\Theta(R)$, assuming that the Sun travels in a Keplerian orbit about the center of the Galaxy.

(b) From your result in part (a), derive analytic expression for the Oort constants A and B. Start from the definition of the constants discussed in the lecture.

(c) Determine numerical values for A and B in the solar neighbourhood, assuming $R_0 = 8$ kpc and $\Theta_0 = 220$ km s^{-1}(orbital radius and velocity of the LSR). Express your answers in units of km s^{-1}kpc^{-1}.

(d) Do your answer in part (c) agree with the measured values for the Milky Way Galaxy? Why or why not?

Problem 4 *Rotation Curve*

Calculate rotation curves $v(r)$ of a thin-disk galaxy.

(a) When the total mass of the galaxy M is located at the center.

(b) When the disk has a finite radius r_d, and the mass of the disk is distributed uniformly with a constant surface density Σ_0.

(c) When the mass is distributed in a disk with the surface density $\Sigma(r) = \dfrac{v_0^2}{2\,\pi G r}$.

(d) In order to have a constant $v(r)$, what kind of surface density distribution is necessary?

(e) In such a disk like (d), how much is the ratio between the disk mass within r_d and $2\,r_d$?