Problem set #9

Problem 1 *Eddington luminosity recap*

We will calculate the Eddinton limit of the accretion onto an AGN.

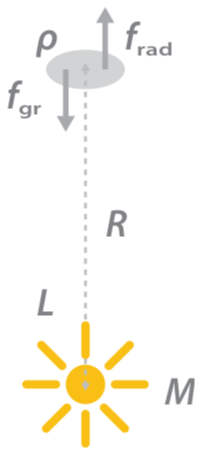

Figure 1

We place a unit volume element of mass at the distance R from the AGN. The mass and the luminosity of the AGN is M and L, respectively. The radiation pressure that the volume element experiences is

$$P_{\rm rad} = \frac{E_{\rm rad}}{c},$$

where $E_{\rm rad}$ is the radiation energy absorbed by the volume element.

(a) Electrons are the main source of the opacity in the ionized gas. Discuss qualitatively why so, by comparing it with the opacity of protons, which are ~1000 times more massive.

(b) Use the cross section of the Thomson scattering by an electron ($\sigma_T = 6.65 \times 10^{-25}\,{\rm cm}^2$), and find that the radiation force acting on the volume element is

$$f_{\rm rad} = \frac{L\,\sigma_T \rho}{4\,\pi R^2 c\, m_p},$$

where m_p is the proton mass and ρ is the mass of the volume element (= mass density). Tell us why we use m_p instead of m_e. Tell us also what happens to the protons when the electrons are blown away by the radiation pressure?

(c) The gravitational attraction that the volume element experiences is

$$f_{\rm gr} = \frac{GM\rho}{R^2}.$$

Check if the unit of $f_{\rm rad}$ is consistent with $f_{\rm gr}$.

(d) Show that the Eddington luminosity is given by

$$L_E = \frac{4\,\pi c\, GM\, m_p}{\sigma_T}$$

(e) Assume that the luminosity of the AGN is $L = 10^{44}\,\mathrm{erg\,s^{-1}}$. Calculate the lower limit of the mass of the central source M_*.

(f) The Schwarzschild radius of the central source is

$$R_S = \frac{2\,GM}{c^2}.$$

Discuss qualitatively what the formula means. How much is the Schwarzschild radius of a solar mass object?

(g) Let a mass element m fall in the central source from $r = \infty$ to R_S. How much energy we can extract at most?

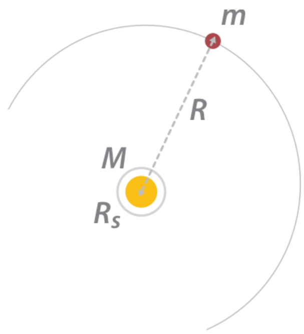

Figure 2

(h) Assume $\eta = 0.1$ of the energy above is converted to the radiation. How much mass accretion is needed per year to maintain the luminosity $L = 10^{44}\,\mathrm{erg\,s^{-1}}$? The mass of the central source is M_* as before. What happens when you accrete more?

Problem 2 *Major mergers of elliptical galaxies*

Elliptical galaxies can theoretically be described as collisionless stellar systems. Explain what we mean by this. Observations and evolutionary models for elliptical galaxies indicate they can grow by mergers with other small and big ellipticals. In this exercise we will use the virial theorem to estimate how the size, the velocity dispersion, and the mean density of an elliptical galaxy will change if it accretes stars.

(a) Assuming the systems have isotropic velocity dispersions and no rotation, give expressions for the total kinetic and gravitational energy (and the total energy) of a galaxy as function of Mass M, stellar mean square velocity $\langle v^2 \rangle$, and effective gravitational radius r_g. (*Hint:* How could an effective radius be suitably defined to describe a galaxy's binding energy?)

Let $\eta = M_\mathrm{a}/M_\mathrm{i}$ be the ratio of the accreted mass M_a to the initial mass M_i (so that the final galaxy has a mass $M_\mathrm{f} = M_\mathrm{i} + M_\mathrm{a} = (1+\eta)M_\mathrm{i}$), and let $\epsilon = \langle v_\mathrm{a}^2 \rangle / \langle v_\mathrm{i}^2 \rangle$ be the ratio of the initial average square speeds to the average square speeds of the accreted material.

(b) What is the energy of the merged galaxy?

(c) How do the velocities, sizes, and densities change when merging two systems? Compute the final-to-initial ratios of these quantities. Give the general expressions as functions of η and ϵ and numerical values for two identical systems.

(d) What happens if you have accretion of smaller systems with $\langle v_\mathrm{a}^2 \rangle \ll \langle v_\mathrm{i}^2 \rangle$? Can the very first galaxies have formed through the mergers of single stars?